Please check the examination details below before entering your candidate information

Candidate surname | Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Time 2 hours

Paper reference **8MA0/01**

Mathematics

Advanced Subsidiary
PAPER 1: Pure Mathematics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 14 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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Turn over

P 6 9 2 0 1 A 0 1 4 8

Pearson

1. Find

$$\int \left(8x^3 - \frac{3}{2\sqrt{x}} + 5\right) dx$$

giving your answer in simplest form.

(4)

Rearrange so that $\sqrt{x}$ is not on the denominator of the fraction. →

$$\int 8x^3 - \frac{3}{2\sqrt{x}} + 5 \, dx \qquad \sqrt{x} = x^{\frac{1}{2}}$$

$$= \int 8x^3 - \frac{3}{2x^{\frac{1}{2}}} + 5 \, dx \qquad \frac{1}{x^a} = x^{-a}$$

$$= \int 8x^3 - \frac{3}{2}x^{-\frac{1}{2}} + 5 \, dx$$

Integrate by adding one to the power, and dividing by the new power.

$$= \frac{8x^4}{4} - \frac{3}{\frac{1}{2} \times 2}x^{\frac{1}{2}} + \frac{5x}{1} + c$$

remember to add c, the constant of integration, because we are integrating without limits.

$$= 2x^4 - 3x^{\frac{1}{2}} + 5x + c$$

Question 1 continued

(Total for Question 1 is 4 marks)

2. $$f(x) = 2x^3 + 5x^2 + 2x + 15$$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$. **(2)**

(b) Find the constants a, b and c such that

$$f(x) = (x + 3)(ax^2 + bx + c)$$

(2)

(c) Hence show that $f(x) = 0$ has only one real root. **(2)**

(d) Write down the real root of the equation $f(x - 5) = 0$ **(1)**

a) The factor theorem states that $f(x-a)$ is a factor of $f(x)$ if $f(a) = 0$.

In this case, $a = -3$, because we are showing $(x+3)$ is a factor.

$\longrightarrow f(-3) = 2(-3)^3 + 5(-3)^2 + 2(-3) + 15$
$= 2(-27) + 5(9) + 2(-3) + 15$
$= -54 + 45 - 6 + 15$

$f(-3) = 0$
$f(-3) = 0$, so $(x+3)$ is a factor

b) To find a, b and c, divide $2x^3 + 5x^2 + 2x + 15$ by $(x+3)$ to find a quadratic.

	$2x^2$	$-x$	5
x	$2x^3$	$-x^2$	$5x$
3	$6x^2$	$-3x$	15

$f(x) = (x+3)(2x^2 - x + 5)$ so $a = 2$, $b = -1$, $c = 5$

c) To show $f(x)$ has only one real root, (-3), show $2x^2 - x + 5$ has no real roots.

to show there are no real roots, find the discriminant. This is the part of the quadratic formula that, if negative, means that there are no real solutions

$\rightarrow b^2 - 4ac = (-1)^2 - (4 \times 2 \times 5)$
$= 1 - 40$

$b^2 - 4ac = -39$
The discriminant is < 0, so the quadratic has no real roots, so $f(x) = 0$ has only 1 real root.

Question 2 continued

d) $f(x-5)$ is a transformation of $f(x)$. It is a translation $\begin{pmatrix}5\\0\end{pmatrix}$, which means the whole graph moves 5 to the right.

The only real root of $f(x)$ is -3. So the only real root of $f(x-5)$ is 2, because the root has shifted 5 co-ordinates to the right.

$f(x-5)=0$ $\quad -3+5=2$

$x=2$ is the only real root of $f(x-5)$

Question 2 continued

Question 2 continued

(Total for Question 2 is 7 marks)

3. The triangle PQR is such that $\overrightarrow{PQ} = 3\mathbf{i} + 5\mathbf{j}$ and $\overrightarrow{PR} = 13\mathbf{i} - 15\mathbf{j}$

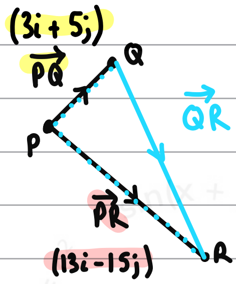
(a) Find $\overrightarrow{QR}$ → end, ↓ start **(2)**

(b) Hence find $|\overrightarrow{QR}|$ giving your answer as a simplified surd. **(2)**

The point S lies on the line segment QR so that $QS : SR = 3 : 2$

(c) Find $\overrightarrow{PS}$ **(2)**

a)



By drawing a diagram, we can find $\overrightarrow{QR}$ by going backwards along $\overrightarrow{PQ}$, and then going along $\overrightarrow{PR}$. So we need to find negative $\overrightarrow{PQ}$ + $\overrightarrow{PR}$

because i and j are perpendicular, you can just add i and j together separately. →

$\overrightarrow{QR} = -\overrightarrow{PQ} + \overrightarrow{PR}$

$= -(3i + 5j) + (13i - 15j)$

$= -3i - 5j + 13i - 15j$

$\overrightarrow{QR} = 10i - 20j$

b) $|\overrightarrow{QR}|$ is the magnitude of $\overrightarrow{QR}$. Use pythagoras to find the magnitude.

$|\overrightarrow{QR}| = \sqrt{(10)^2 + (-20)^2}$

$= \sqrt{100 + 400}$

$= \sqrt{500}$

$= \sqrt{100}\sqrt{5}$

$= 10\sqrt{5}$

pythagoras: $a^2 = b^2 + c^2$

$a = \sqrt{b^2 + c^2}$

where a is the magnitude we are trying to find.

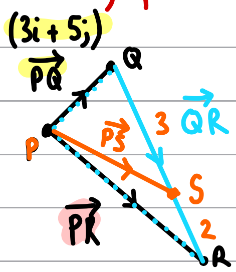
$b = 10(i)$ $c = -20(j)$

To find in its simplest form, find the largest square number which is a factor of 500, (100) →

c) To find $\overrightarrow{PS}$, find $\overrightarrow{PQ} + \overrightarrow{QS}$. Because $QS : SR = 3:2$, $\overrightarrow{QS} = 3/5$ of $\overrightarrow{QR}$

$\overrightarrow{QS} = \frac{3}{3+2}\overrightarrow{QR}$

$\overrightarrow{QS} = \frac{3}{5}\overrightarrow{QR}$



$\overrightarrow{PS} = \overrightarrow{PQ} + \frac{3}{5}(\overrightarrow{QR})$

$= 3i + 5j + \frac{3}{5}(10i - 20j)$

$= 3i + 5j + \frac{30}{5}i - \frac{60}{5}j$

$= 3i + 5j + 6i - 12j$

$\overrightarrow{PS} = 9i - 7j$

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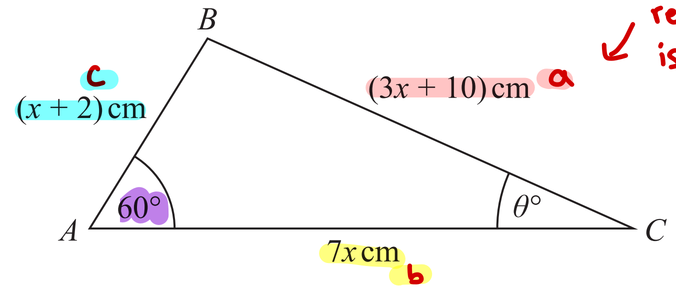
Question 3 continued

(Total for Question 3 is 6 marks)

4.



remember side a is opposite to angle A

Figure 1

Figure 1 shows a sketch of triangle ABC with $AB = (x + 2)$ cm, $BC = (3x + 10)$ cm, $AC = 7x$ cm, angle $BAC = 60°$ and angle $ACB = \theta°$

(a) (i) Show that $17x^2 - 35x - 48 = 0$ **(3)**

(ii) Hence find the value of x. **(1)**

(b) Hence find the value of θ giving your answer to one decimal place. **(2)**

a) We need to use the cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

$$(3x+10)^2 = (7x)^2 + (x+2)^2 - 2(7x)(x+2)\cos 60$$
$$9x^2 + 60x + 100 = 49x^2 + x^2 + 4x + 4 - (14x^2 + 28x)\cos 60$$
$$9x^2 + 60x + 100 = 49x^2 + x^2 + 4x + 4 - (7x^2 + 14x)$$
$$9x^2 + 60x + 100 = 49x^2 + x^2 + 4x + 4 - 7x^2 - 14x$$
$$0 = 34x^2 - 70x - 96$$
$$0 = 17x^2 - 35x - 48$$

we know $\cos 60 = \frac{1}{2}$

put all terms on one side and divide by 2 to get the required form.

i) solve $17x^2 - 35x - 48 = 0$ to find x

$$0 = (17x+16)(x-3)$$
$$17x + 16 = 0 \qquad x + 3 = 0$$
$$17x = -16 \qquad x = -3$$
$$x = \frac{-16}{17}$$

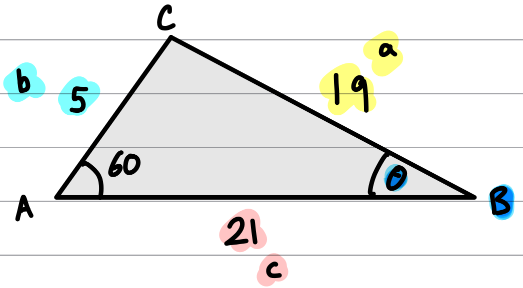
It is factorisable, so factorise to find x

$x = 3$

(x must be positive because a side of a triangle cannot have a negative length)

Question 4 continued

b) we can use the cosine rule again: $a^2 = b^2 + c^2 - 2bc\cos A$



We are trying to find angle B, (B = θ) so swap a and b around in the equation

B = θ

$b^2 = a^2 + c^2 - 2ac\cos B$

$5^2 = 19^2 + 21^2 - 2 \times 19 \times 21 \times \cos B$

$25 = 802 - 798\cos B$

$798\cos B = 777$

$\cos B = 0.97368....$

$\theta = B = 13.1735...$

$\theta = 13.2°$ (1dp)

we know $x = 3$, so sub in $x = 3$ to find the side lengths.

Turn over

P 6 9 2 0 1 A 0 1 1 4 8

Question 4 continued

Question 4 continued

(Total for Question 4 is 6 marks)

5. The mass, A kg, of algae in a small pond, is modelled by the equation

$$A = pq^t$$

where p and q are constants and t is the number of weeks after the mass of algae was first recorded.

Data recorded indicates that there is a linear relationship between t and $\log_{10} A$ given by the equation

$$\log_{10} A = 0.03t + 0.5$$

(a) Use this relationship to find a complete equation for the model in the form

$$A = pq^t$$

giving the value of p and the value of q each to 4 significant figures.

(4)

(b) With reference to the model, interpret

(i) the value of the constant p,

(ii) the value of the constant q.

(2)

(c) Find, according to the model,

(i) the mass of algae in the pond when $t = 8$, giving your answer to the nearest 0.5 kg,

(ii) the number of weeks it takes for the mass of algae in the pond to reach 4 kg.

(3)

(d) State one reason why this may not be a realistic model in the long term.

(1)

We can get rid of the $\log_{10}$ by raising all terms to the power of 10. This is because $10^{\log_{10} A} = A$

$\log_{10} A = 0.03t + 0.5$

$10^{\log_{10} A} = A$

$10^{\log_{10} A} = 10^{0.03t + 0.5}$

$A = 10^{0.03t + 0.5}$

$x^{a+b} = x^a \times x^b$ so $10^{0.03t + 0.5} = 10^{0.03t} \times 10^{0.5}$

$A = 10^{0.03t} \times 10^{0.5}$

looking at the form $A = pq^t$, we need to change $10^{0.03t}$ to $(10^{0.03})^t$. Use the power rule $(x^a)^b = x^{ab}$

$A = (10^{0.03})^t \times 10^{0.5}$

$A = 10^{0.5} \times (10^{0.03})^t$

$A = p\,q^t$

now it is in the form $A = pq^t$, we can find the values of p and q

$p = 10^{0.5} = 3.162$

$q = 10^{0.03} = 1.072$

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Made with Goodnotes

P 6 9 2 0 1 A 0 1 4 4 8

Question 5 continued

The complete equation is: $A = 3.162 \times 1.072^t$

bi) When $t = 0$, $q^t = 1$, so $A = p$. Therefore p is the original mass of algae (mass when $t=0$).

ii) q is the rate of growth of algae. Because t is measured in weeks and A is in kilograms, it is the rate of growth in kilograms each week.

c) i) Sub in $t = 8$ into $A = 3.162 \times 1.072^t$

$A = 3.162 \times 1.072^8$

$A = 3.162 \times 1.744$

$A = 5.51$

$A = 5.5$ kg (to the nearest 0.5 kg)

ii) Sub in $A = 4$ to find t.

divide by 3.162

$4 = 3.162 \times 1.072^t$

$1.265 = 1.072^t$

take logs of base 1.072 of both sides

$\log_{1.072} 1.265 = \log_{1.072} 1.072^t$

$\log_{1.072} 1.072^t = t$

$\log_{1.072} 1.265 = t$

$t = 3.38$

$= 3.4$ weeks

d) the model predicts unlimited growth, which is unlikely

Question 5 continued

Question 5 continued

(Total for Question 5 is 10 marks)

6. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(3 - \frac{2x}{9}\right)^8$$

giving each term in simplest form.

(4)

can use Pascal's triangle or NCR function on calculator.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

$$f(x) = \left(\frac{x-1}{2x}\right)\left(3 - \frac{2x}{9}\right)^8$$

(b) Find the coefficient of x^2 in the series expansion of f(x), giving your answer as a simplified fraction.

(2)

a) $\left(3 - \frac{2x}{9}\right)^8 = {}^8C_0 \times 3^8 \times \left(-\frac{2x}{9}\right)^0 + {}^8C_1 \times 3^7 \times \left(-\frac{2x}{9}\right)^1 + {}^8C_2 \times 3^6 \times \left(-\frac{2x}{9}\right)^2 + {}^8C_3 \times 3^5 \times \left(-\frac{2x}{9}\right)^3$

$= 1 \times 6561 \times 1 + 8 \times 2187 \times -\frac{2x}{9} + 28 \times 729 \times \frac{4x^2}{81} + 56 \times 243 \times -\frac{8x^3}{729}$

$= 6561 - 3888x + 1008x^2 - \frac{448}{3}x^3$

b) split up the first fraction into two fractions

$\frac{x-1}{2x} = \frac{x}{2x} - \frac{1}{2x} = \frac{1}{2} - \frac{1}{2x}$

$f(x) = \left(\frac{1}{2} - \frac{1}{2x}\right)\left(3 - \frac{2x}{9}\right)^8$

So, we must multiply $\frac{1}{2}$ by the x^2 term in the second bracket, and $-\frac{1}{2x}$ by the x^3 term in the second bracket, to get two terms in x^2.

$f(x) = \left(\frac{1}{2} - \frac{1}{2x}\right)\left(6561 - 388x + 1008x^2 - \frac{448}{3}x^3 + \ldots\right)$

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Question 6 continued

	$1008x^2$	$-\frac{448}{3}x^3$
$\frac{1}{2}$	$504x^2$	$-\frac{244}{3}x^3$
$-\frac{1}{2x}$	$-504x$	$\frac{244}{3}x^2$

← both of these terms are coefficents of x^2

so the total coefficient of x^2 is $504 + \frac{244}{3} = \frac{1736}{3}$

coefficient of $x^2 = \boxed{\frac{1736}{3}}$

Question 6 continued

Question 6 continued

(Total for Question 6 is 6 marks)

7. (a) Factorise completely $9x - x^3$

(2)

The curve C has equation

$$y = 9x - x^3$$

(b) Sketch C showing the coordinates of the points at which the curve cuts the x-axis.

(2)

The line l has equation $y = k$ where k is a constant.

Given that C and l intersect at 3 distinct points,

(c) find the range of values for k, writing your answer in set notation.

Solutions relying on calculator technology are not acceptable.

(3)

a) x is a common factor

$9x - x^3$

$x(9 - x^2)$

$x(3 + x)(3 - x)$

in $(9 - x^2)$, both 9 and x^2 are square numbers, so you can factorise the difference of two squares

$(a^2 - b^2) = (a + b)(a - b)$

so $(9 - x^2) = (3 + x)(3 - x)$

so $9x - x^3 = x(3 + x)(3 - x)$

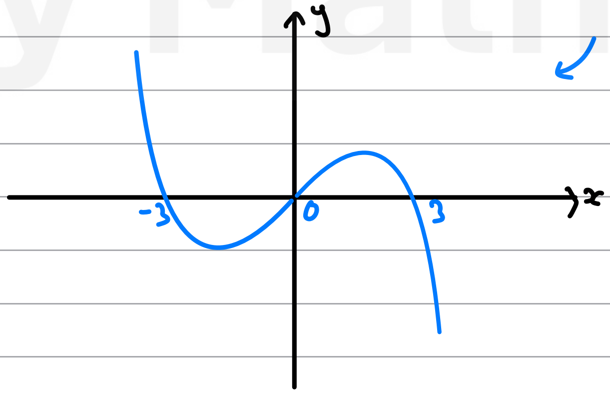
b) To find roots set $y = x(3+x)(3-x)$ equal to 0

$0 = x(3 + x)(3 - x)$

$x = 0$ $3 + x = 0$ $3 - x = 0$

$x = -3$ $x = 3$

roots are $0, -3, 3$



the graph is a cubic, so has a shape ∼ or ∽ shape.

Because the coefficient of x^3 is -1, the shape of the graph is a negative cubic, a ∽ shape

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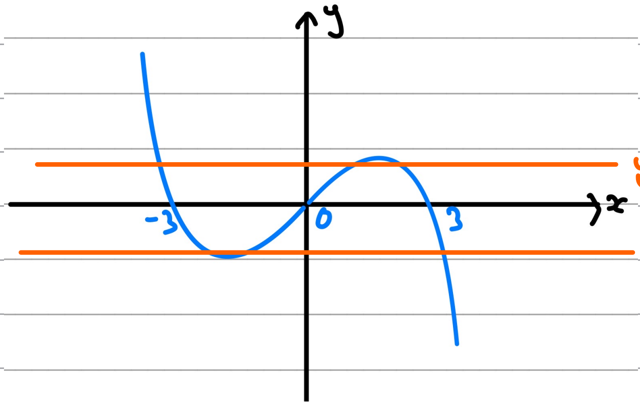
Question 7 continued

c)



greatest value of k just less than the peak

smallest value of k just greater than the trough.

We can find these points on the peaks and troughs, because they are turning points - the gradient = 0

find gradient by differentiating y. Bring the power down, then subtract 1 from the power.

$$y = 9x - x^3$$

$$\frac{dy}{dx} = 9 - 3x^2$$

$$0 = 9 - 3x^2$$

$$0 = x^2 - 3$$

$$0 = (x - \sqrt{3})(x + \sqrt{3})$$

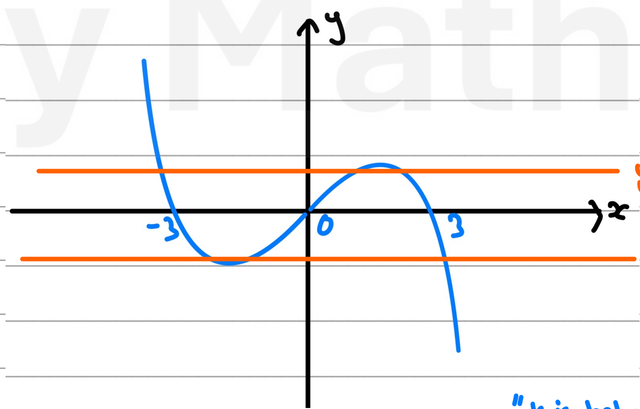
$$x = \sqrt{3}, -\sqrt{3}$$

set the gradient equal to 0. Then solve to find x at the turning points

sub back in x to find their y values.

$$y = 9\sqrt{3} - (\sqrt{3})^3 \qquad y = -9\sqrt{3} - (-\sqrt{3})^3$$

$$y = 6\sqrt{3} \qquad y = -6\sqrt{3}$$



So the value of k must be between these two values to give 3 distinct solutions.

"k is a member of real numbers"

"k is between $-6\sqrt{3}$ and $6\sqrt{3}$"

$$\{k \in \mathbb{R} : -6\sqrt{3} < k < 6\sqrt{3}\}$$

Question 7 continued

Question 7 continued

(Total for Question 7 is 7 marks)

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The air pressure, P kg/cm^2, inside a car tyre, t minutes from the instant when the tyre developed a puncture is given by the equation

$$P = k + 1.4e^{-0.5t} \qquad t \in \mathbb{R} \qquad t \geqslant 0$$

where k is a constant.

Given that the initial air pressure inside the tyre was 2.2 kg/cm^2

(a) state the value of k.

(1)

From the instant when the tyre developed the puncture,

(b) find the time taken for the air pressure to fall to 1 kg/cm^2
Give your answer in minutes to one decimal place.

(3)

(c) Find the rate at which the air pressure in the tyre is decreasing exactly 2 minutes from the instant when the tyre developed the puncture.
Give your answer in kg/cm^2 per minute to 3 significant figures.

(2)

a) Initially, $t=0$, so $1.4e^{-0.5t} = 1.4$ (because anything to the 0 equals 1).
Initially $P = 2.2$
@ $t=0$ $\quad 2.2 = k + 1.4$
$k = 0.8$

b) $P = 0.8 + 1.4e^{-0.5t}$
$1 = 0.8 + 1.4e^{-0.5t}$ ← set $p=1$ to find t
$0.2 = 1.4e^{-0.5t}$
$1/7 = e^{-0.5t}$ ← divide both sides by 1.4 to get $e^{-0.5t}$ by itself.
$\ln 1/7 = \ln e^{-0.5t}$ ← take ln of both sides
$\ln 1/7 = -0.5t$ ← $\ln e^{x} = x$, so $\ln e^{-0.5t} = -0.5t$
$t = -2\ln 1/7$ ← rearrange to find t.
$t = 3.89$
$t = 3.9$ minutes (2sf)

c) to find the rate, we differentiate - they are synonymous

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Question 8 continued

$p = 0.8 + 1.4e^{-0.5t}$

$\frac{dp}{dt} = -0.5 \times 1.4e^{-0.5t}$

$= -0.7e^{-0.5t}$

remember, when we differentiate e, we bring down the derivative (-0.5) of the power, and we don't change the power.

sub in $t = 2$

@$t=2$ $\frac{dp}{dt} = -0.7e^{-0.5\times 2}$

$= -0.7e^{-1}$

$= -0.2575...$

so the air pressure is decreasing at a rate of 0.258 kg/cm^2 per min

(Total for Question 8 is 6 marks)

9. (a) Given that $p = \log_3 x$, where $x > 0$, find in simplest form in terms of p,

(i) $\log_3\left(\frac{x}{9}\right)$

(ii) $\log_3\left(\sqrt{x}\right)$

(2)

(b) Hence, or otherwise, solve

$$2\log_3\left(\frac{x}{9}\right) + 3\log_3\left(\sqrt{x}\right) = -11$$

giving your answer as a simplified fraction.

Solutions relying on calculator technology are not acceptable.

(4)

a) i) use rule: $\log a - \log b = \log \frac{a}{b}$

$\log_3\left(\frac{x}{9}\right) = \log_3 x - \log_3 9$ ← $\log_3 9 = 2$ because $3^2 = 9$

$\log_3\left(\frac{x}{9}\right) = p - 2$

ii) use rule $\log a^x = x\log a$

$\log_3\sqrt{x} = \log_3 x^{1/2}$ ← $\sqrt{x} = x^{1/2}$

$= \frac{1}{2}\log_3 x$

$\log_3\sqrt{x} = \frac{1}{2}p$

b) sub in $\log_3\left(\frac{x}{9}\right) = p - 2$ and $\log_3\sqrt{x} = \frac{1}{2}p$

$2\log_3\left(\frac{x}{9}\right) + 3\log_3\sqrt{x} = -11$

$2(p-2) + 3\left(\frac{1}{2}p\right) = -11$

$2p - 4 + \frac{3}{2}p = -11$

$\frac{7}{2}p = -7$

$p = -2$

sub in $p = -2$ into $p = \log_3 x$

Question 9 continued

$-2 = \log_3 x$

$x = 3^{-2}$

$x = \frac{1}{9}$

(Total for Question 9 is 6 marks)

10.

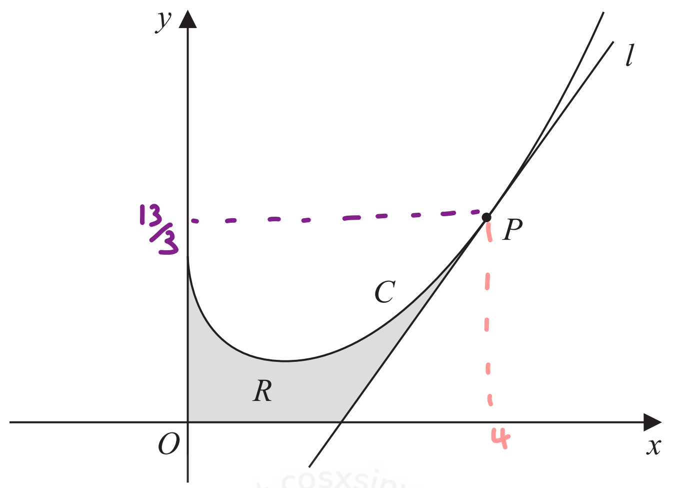


Figure 2

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 2 shows a sketch of part of the curve C with equation

$$y = \frac{1}{3}x^2 - 2\sqrt{x} + 3 \qquad x \geqslant 0$$

The point P lies on C and has x coordinate 4

The line l is the tangent to C at P.

(a) Show that l has equation

$$13x - 6y - 26 = 0$$

(5)

The region R, shown shaded in Figure 2, is bounded by the y-axis, the curve C, the line l and the x-axis.

(b) Find the exact area of R.

(5)

a) We have to find the gradient of l. We can do this by finding the gradient of C at $x=4$, by differentiating $y = \frac{1}{3}x^2 - 2\sqrt{x} + 3$

$\sqrt{x} = x^{1/2}$

$y = \frac{1}{3}x^2 - 2x^{1/2} + 3$

$\frac{dy}{dx} = 2\times\frac{1}{3}x - 2\times\frac{1}{2}x^{-1/2}$

$= \frac{2}{3}x - x^{-1/2}$

differentiate by bringing down the power and subtracting 1 from the power

@ $x=4$ $\quad \frac{dy}{dx} = \frac{2}{3}\times 4 - 4^{-1/2}$ $\quad 4^{-1/2} = \frac{1}{2}$

gradient $= 13/6$

Question 10 continued

Now find the y co-ordinate of P by subbing in $x=4$ into the original equation.

@ $x=4$ $\quad y=\frac{1}{3}\times 4^2 - 2\sqrt{4} + 3$

$= 16/3 - 4 + 3$

$= 13/3$

$P = (4, 13/3)$

Find the equation of L by subbing in $m = 13/6$, $x = 4$, $y = 13/3$ into $y = mx + c$ to find c.

$y = mx + c$

$13/3 = 13/6 \times 4 + c$

$13/3 = 26/3 + c$

$c = -13/3$

$y = \frac{13}{6}x - \frac{13}{3}$

$6y = 13x - 26$

$13x - 6y - 26 = 0$

b)

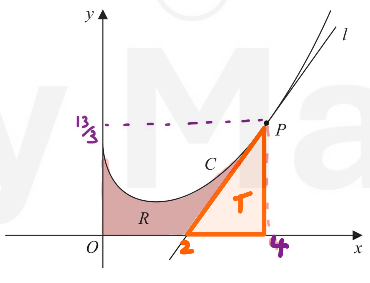


Figure 2

To find R, integrate the curve C between $x=4$ and $x=0$. Then subtract triangle T

To find T, use area = ½ × base × height

height = 13/3

sub in $y=0$ into $13x - 6y - 26 = 0$ to find the base

$13x - 26 = 0$

$13x = 26$

$x = 2$ $\quad$ base $= 4 - 2 = 2$

integrate by adding 1 to the power and dividing by the new power.

$R = \int_0^4 \left(\frac{1}{3}x^2 - 2x^{1/2} + 3\right) dx - T$

$R = \left[\frac{1}{9}x^3 - \frac{4}{3}x^{3/2} + 3x\right]_0^4 - \frac{1}{2}\times \text{base} \times \text{height}$

Sub in the limits 4 and 0.

$R = \left(\frac{1}{9}\times 4^3 - \frac{4}{3}\times 4^{3/2} + 3\times 4\right) - \left(\frac{1}{9}\times 0^3 - \frac{4}{3}\times 0^{3/2} + 3\times 0\right) - \frac{1}{2}\times 2 \times \frac{13}{3}$

Question 10 continued

$R = \left(\frac{64}{9} - \frac{32}{3} + 12\right) - 0 - \frac{1}{2} \times 2 \times \frac{13}{3}$

$= \frac{76}{9} - \frac{13}{3}$

$R = \frac{37}{9}$

$R = \frac{37}{9}$

Question 10 continued

(Total for Question 10 is 10 marks)

11.

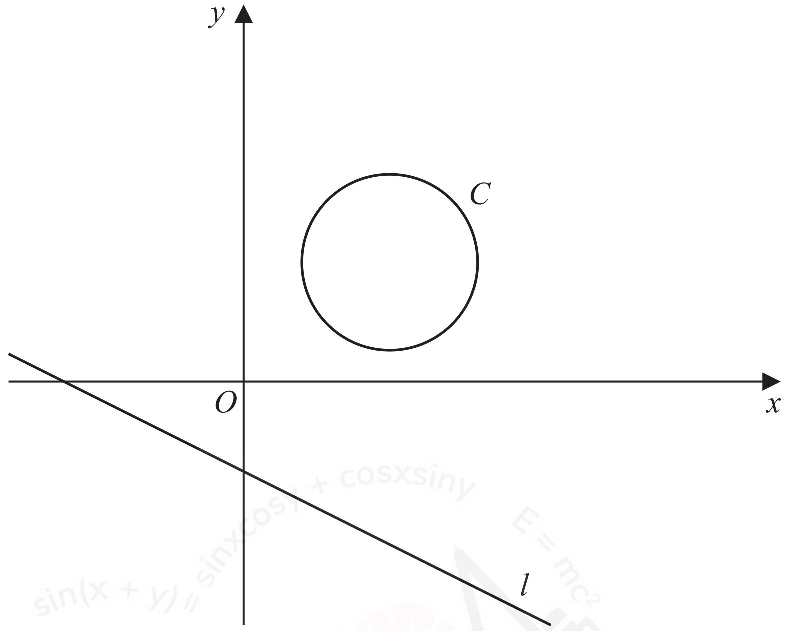


Figure 3

Figure 3 shows the circle C with equation

$$x^2 + y^2 - 10x - 8y + 32 = 0$$

and the line l with equation

$$2y + x + 6 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the radius of C.

(3)

(b) Find the shortest distance between C and l.

(5)

a) Complete the square of both x and y to get the equation into the form $(x-a)^2 + (y-b)^2 = r^2$, where (a, b) is the centre and r is the radius

$$x^2 + y^2 - 10x - 8y + 32 = 0$$
$$(x-5)^2 - 25 + (y-4)^2 - 16 + 32 = 0$$
$$(x-5)^2 + (y-4)^2 - 9 = 0$$
$$(x-5)^2 + (y-4)^2 = 9$$
$$(x-a)^2 + (y-b)^2 = r^2$$

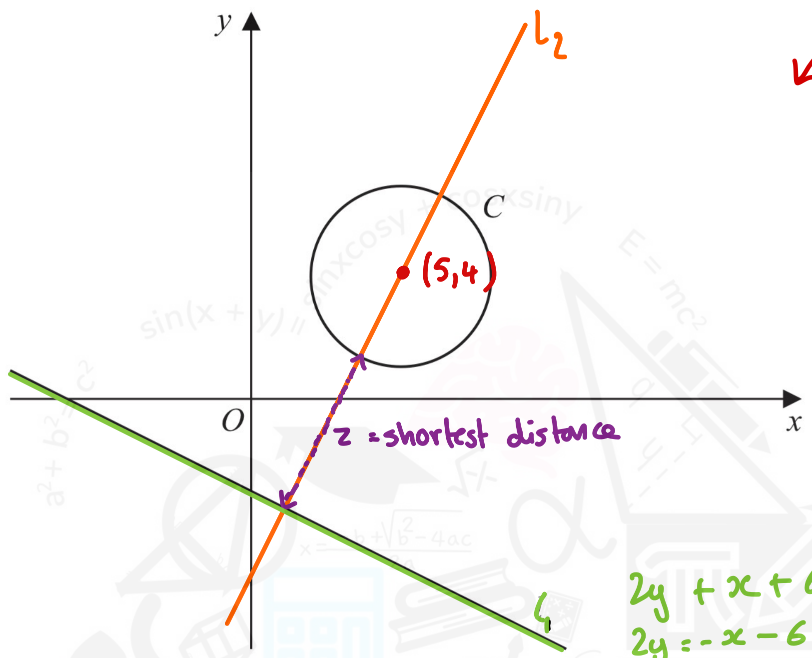
P 6 9 2 0 1 A 0 3 4 4 8

Question 11 continued

centre $=(a,b)$ | centre $=(5,4)$

radius $= r$ | radius $=\sqrt{9}=3$

b)



the shortest distance is perpendicular to line l_1 and its line would pass through the centre of the circle.

$2y + x + 6 = 0$

$2y = -x - 6$

$y = -\frac{1}{2}x - 3$

rearrange l_1 to get it in the form $y = mx + c$

First, find l_2. It is perpendicular to l_1, so their gradients multiply to get -1.

gradient of $l_1 = -\frac{1}{2}$, so gradient of $l_2 = 2$ because $-\frac{1}{2} \times 2 = -1$

$y = mx + c$

sub in (5,4) $4 = 2 \times 5 + c$

$4 = 10 + c \quad c = -6$

equation of $l_2 = y = 2x - 6$

Find the point of intersection between l_1 and l_2, by setting them equal to one another.

$y = -\frac{1}{2}x - 3 \quad y = 2x - 6 \qquad -\frac{1}{2}x - 3 = 2x - 6$

$3 = \frac{5}{2}x$

$x = \frac{6}{5}$

$y = -\frac{18}{5}$

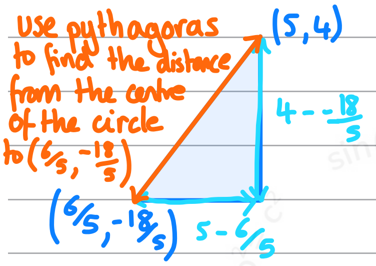
Turn over

Question 11 continued

point of intersection $= \left(\frac{6}{5}, -\frac{18}{5}\right)$

If we find the distance between the point of intersection of l_1 and l_2, and the centre of the circle $(5,4)$, we can subtract the radius (3) to find the shortest distance.

shortest distance = distance between $\left(\frac{6}{5}, -\frac{18}{5}\right)$ and $(5,4)$ − radius



$$= \sqrt{\left(5 - \frac{6}{5}\right)^2 + \left(4 - -\frac{18}{5}\right)^2} - 3$$

$$= \sqrt{\left(\frac{19}{5}\right)^2 + \left(\frac{38}{5}\right)^2} - 3$$

$$= \frac{19\sqrt{5}}{5} - 3$$

shortest distance $= \frac{19\sqrt{5} - 15}{5}$

Question 11 continued

(Total for Question 11 is 8 marks)

12. A company makes drinks containers out of metal.

The containers are modelled as closed cylinders with base radius r cm and height h cm and the capacity of each container is 355 cm^3

The metal used

- for the circular base and the curved side costs 0.04 pence/cm^2
- for the circular top costs 0.09 pence/cm^2

Both metals used are of negligible thickness.

(a) Show that the total cost, C pence, of the metal for one container is given by

$$C = 0.13\pi r^2 + \frac{28.4}{r}$$

(4)

(b) Use calculus to find the value of r for which C is a minimum, giving your answer to 3 significant figures.

(4)

(c) Using $\frac{d^2C}{dr^2}$ prove that the cost is minimised for the value of r found in part (b).

(2)

(d) Hence find the minimum value of C, giving your answer to the nearest integer.

(2)

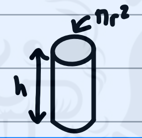
a) Find h in terms of r, because there is no h in the equation we are trying to find.

volume of container = 355

$\pi r^2 h = 355$

$h = \frac{355}{\pi r^2}$

volume of cylinder $= \pi r^2 h$



Find the cost of the base sides and top.

area of base (circle): πr^2

cost of base $= 0.04 \times \pi r^2$

$= 0.04\pi r^2$

area of curved side $= 2\pi r h = 2\pi r\left(\frac{355}{\pi r^2}\right) = \frac{670}{r}$

cost of curved side $= 0.04 \times \frac{710}{r}$

$= \frac{142}{5r} = \frac{28.4}{r}$

area of top (circle) $= \pi r^2$

cost of top $= 0.09 \times \pi r^2$

$= 0.09\pi r^2$

Question 12 continued

total cost $= 0.04\pi r^2 + \frac{28.4}{r} + 0.09\pi r^2$

$= 0.13\pi r^2 + \frac{28.4}{r}$

b) To find a minimum (or maximum), we have to differentiate.
Differentiate C with respect to r

$C = 0.13\pi r^2 + \frac{28.4}{r}$ — $\frac{1}{r} = r^{-1}$

$C = 0.13\pi r^2 + 28.4r^{-1}$

differentiate by bringing the power down, and subtracting 1 from the power.

$\frac{dC}{dr} = 2\times 0.13\pi r + -1\times 28.4r^{-2}$

$\frac{dC}{dr} = 0.26\pi r - 28.4r^{-2}$ — $r^{-2} = \frac{1}{r^2}$

$= 0.26\pi r - \frac{28.4}{r^2}$

to find the minimum, set $\frac{dC}{dr}$ equal to 0 (like if you were trying to find a stationary point on a graph.

$\frac{dC}{dr} = 0.26\pi r - \frac{28.4}{r^2} = 0$

$0.26\pi r^3 - 28.4 = 0$

$0.26\pi r^3 = 28.4$

$r^3 = \frac{28.4}{0.26\pi}$

$r = 3.26$ cm

c) To find $\frac{d^2C}{dr^2}$, differentiate again.

$\frac{dC}{dr} = 0.26\pi r - 28.4r^{-2}$

differentiate by bringing down the power, and subtracting 1 from the power.

$\frac{d^2C}{dr^2} = 0.26\pi + 56.8r^{-3}$

Question 12 continued

Sub in $r = 3.26$ $\quad \frac{d^2C}{dr^2} = 0.26\pi + 56.8 \times 3.26^{-3}$

$= 0.26\pi + 1.64$

the second derivative is greater than 0. $\rightarrow = 2.5$ (2sf) $\quad 2.5 > 0$

A positive second derivative means that it is a minimum

d) sub in $r = 3.26$ in $C = 0.13\pi r^2 + \frac{28.4}{r}$, because we have found the value of r already (3.26) which gives the minimum value of C in part b.

$C = 0.13\pi(3.26)^2 + \frac{28.4}{3.26}$

$= 4.34 + 8.71$

$= 13.1$

$C = 13p$

Question 12 continued

(Total for Question 12 is 12 marks)

13. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{1}{\cos\theta} + \tan\theta \equiv \frac{\cos\theta}{1-\sin\theta} \qquad \theta \neq (2n+1)90° \quad n \in \mathbb{Z}$$

(3)

Given that $\cos 2x \neq 0$

(b) solve for $0 < x < 90°$

$$\frac{1}{\cos 2x} + \tan 2x = 3\cos 2x$$

giving your answers to one decimal place.

(5)

a) $\dfrac{1}{\cos\theta} + \tan\theta$ ①

$= \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$ ①

① $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{1+\sin\theta}{\cos\theta}$

multiply top and bottom by $(1-\sin\theta)$

$= \dfrac{1+\sin\theta\,(1-\sin\theta)}{\cos\theta(1-\sin\theta)}$

$= \dfrac{1-\sin^2\theta}{\cos\theta(1-\sin\theta)}$ ②

② $\sin^2\theta + \cos^2\theta = 1$

$1-\sin^2\theta = \cos^2\theta$

$= \dfrac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$ ②

cos θ is common to the numerator and denominator.

$= \dfrac{\cos\theta}{1-\sin\theta}$

b) Comparing the first and second set of equations, we see $\theta = 2x$.

$\therefore \dfrac{1}{\cos 2x} + \tan 2x = \dfrac{\cos 2x}{1-\sin 2x} = 3\cos 2x$

solve to find x → $\dfrac{\cos 2x}{1-\sin 2x} = 3\cos 2x$

Question 13 continued

$\cos 2x = 3\cos 2x(1 - \sin 2x)$

common factor of $\cos 2x$

$3\cos 2x(1-\sin 2x) - \cos 2x = 0$

$\cos 2x(3(1-\sin 2x) - 1) = 0$

$\cos 2x(3 - 3\sin 2x - 1) = 0$

$\cos 2x(2 - 3\sin 2x) = 0$

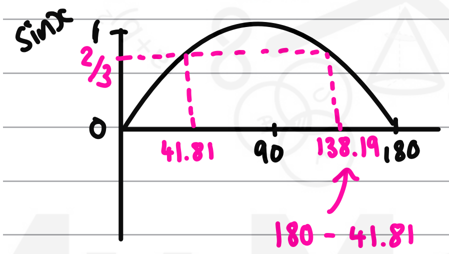
$\cos 2x = 0$ $2 - 3\sin 2x = 0$

in the question $\cos 2x \neq 0$

$2 = 3\sin 2x$

$\sin 2x = 2/3$

because the domain is $0 < x < 90$, but we are solving $\sin 2x = \frac{2}{3}$ we convert the domain to $0 < 2x < 180$



use the graph to help find the second value of $2x$

$\sin 2x = 2/3$

$2x = 41.81..$

$= 138.19$

$x = 20.9°$

$x = 69.1°$

DO NOT WRITE IN THIS AREA

P 6 9 2 0 1 A 0 4 3 4 8

Question 13 continued

P 6 9 2 0 1 A 0 4 4 4 8

Question 13 continued

(Total for Question 13 is 8 marks)

14. (i) A student states

"if x^2 is greater than 9 then x must be greater than 3"

Determine whether or not this statement is true, giving a reason for your answer.

(1)

(ii) Prove that for all positive integers n,

$$n^3 + 3n^2 + 2n$$

is divisible by 6

(3)

a) The statement is not true, because the square of negative numbers below −3 is greater than 9.

ii) $n^3 + 3n^2 + 2n$ ↓ take out common factor n

$= n(n^2 + 3n + 2)$ ↓ factorise the quadratic

$= n(n+2)(n+1)$

$= n(n+1)(n+2)$

↑ this is the product of 3 consecutive integers, so it must include 1 number which is a multiple of 3 and atleast 1 even number.

As $n(n+1)(n+2)$ is a multiple of 2 and 3, it must be a multiple of 6, so $n^3 + 3n^2 + 2n$ is a multiple of 6.

Question 14 continued

Question 14 continued

(Total for Question 14 is 4 marks)

TOTAL FOR PAPER IS 100 MARKS